A Study of Form:
Mutually Supported Stick Structures

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INTRODUCTION

An interesting aspect of structures is the particular way in which short elements combine together to create an assemblage capable of spanning a distance much greater than the length of any of the individual elements. A truss is an obvious example of this kind of structural logic. But what are the specific characteristics that define the class of structures to which the truss belongs? In a pure sense, a truss is a structure with a triangulated pattern of pin connected, axial force members in which all supported loads are applied exclusively at the joints. This description identifies the characteristics of the type, enabling us to group together a diverse range of structural forms whose geometry and other features fit the criteria. In this way we use morphology to categorize known structures that all share common attributes. On the other hand, it is possible for morphology to lead to the *discovery* of new structures by predicting the existence of forms based on the generation of a range of parametric combinations.

This paper examines a special class of structures which, like the truss, are composed of short elements forming an assemblage that can span a greater distance than the length of the largest individual component. But unlike the truss these structures do not rely on connections at the joints to transfer loads, and the relationship between individual elements is characterized by a unique condition of mutuality.

Several authors[1] have examined structures similar in nature which are referred to in the literature as *reciprocal frames*[2]. However, the distinction between structures with or without connections is unclear and generally not discussed. For example, regarding the beam framing system proposed by Serlio in Book 1 of *The Five Books of Architecture* written between 1537 and 1547[3], mentioned in a paper by Melaragno[4], the role of the connectors, which are visible in the drawing, is not mentioned, nor are certain geometrical issues regarding the assembly of the framing examined. Our own investigation was triggered by a four beam structure made by students on the occasion of a built project shown in figure 2, after which we came across these other studies.

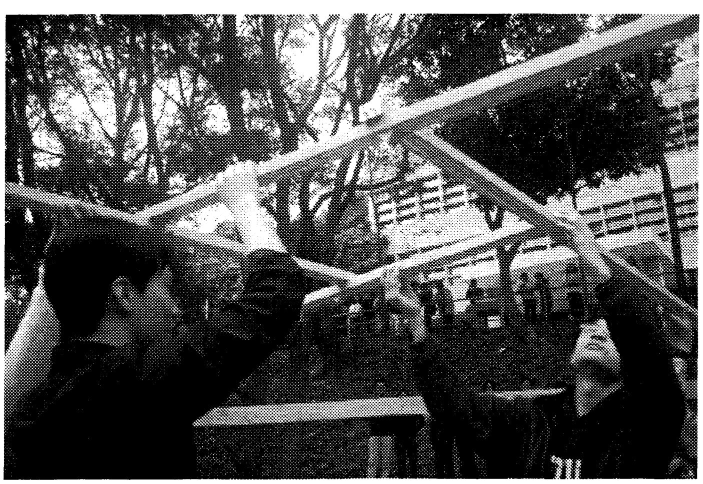

Figure 1: Lever beams in student project

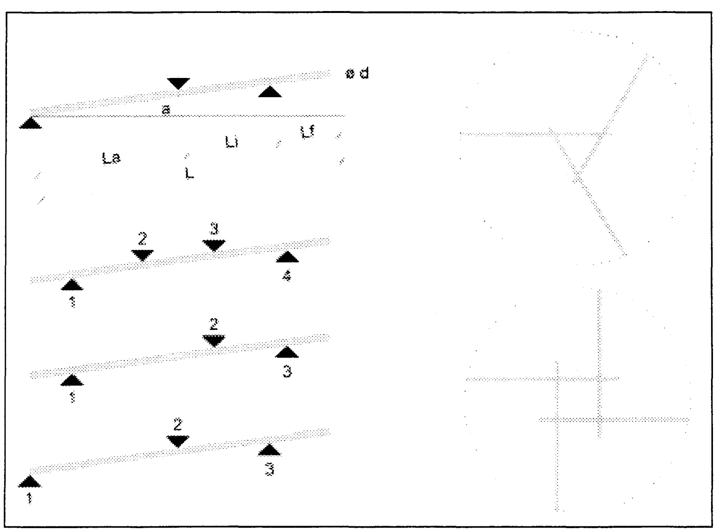

Figure 2: Lever beam variations and basic units

Aside from Serlio, other examples of suspended beam structures have appeared throughout history and have been studied by David Yeomans[5]. There are also a number of built contemporary works. These are primarily three dimensional roof structures, the best known among them being the timber roof structure of the Seiwa Bunraku Puppet Theatre in Kumamoto Prefecture of southern Japan, designed by Kazuhiro Ishii[6]. Ishii has also designed several private residences using the reciprocal frame principle, such as the Enomoto residence[7].

DESCRIPTION OF LEVER BEAM STRUCTURES

Within the class of mutually supporting stick structures, to which reciprocal frames also belong, we examine more closely a subclass which we will refer to as *lever beam structures*. Mutually supporting stick structures can span longer distances than their individual elements acting alone, and fulfil the requirement that their elements both support and be supported by others at the same time. Lever beam structures have to fulfil the additional requirement that no connectors are used to transfer loads, although connectors might be used for keeping elements in position. We call this subclass lever beam structures because the lever beam best describes the structural behaviour of an individual element.

Figure 2 illustrates the lever beam which is the principle mechanism of load transfer and the basic element underlying all the structures in this class. The lever beam is a straight, rigid element with one end resting on the ground (1) and the other supported by a beam (3). Somewhere in between the beam supports another beam (2). The beam is in equilibrium if the clockwise moment created by the weight of the beam being supported at (2) is balanced by a counter moment caused by the reaction of the supporting beam at (3). The closer that the supported beam (2) is to (3), the larger will be the portion of its load that is transferred to the beam at (3).

This relationship of forces can be described by several geometrical parameters which are listed below. The values of these parameters determine much of the variation in form of these structures, independent from the actual cross- and longitudinal sections of the beam. The important role of the beam section will be investigated in a future study. For now we consider only straight beams with a circular cross section.

GEOMETRICAL PARAMETERS:

Diameter	d
Beam Length	$L (= L_A + L_I + L_F)$
Anchor Length	L_A
Interior Length	L_I
Free Length	L_F
Angle of inclination	a

We explore the question of possible forms which lever beam structures can take on two levels. First we look at the formation of units and then how such units can be expanded to create larger structures. The smallest lever beam structure is formed by two beams which support each other. This can only be done with non straight beams, or with straight beams and connectors. The smallest structure which falls into the limitations of this study consists of three beams. We consider this as a possible basic unit shown in figure 2. Other basic units can be formed by any number of beams, characterised by a regular polygon in the centre of the structure having the same number of sides as it has beams, as illustrated in figure 4 top. Non symmetrical units can also be formed. For the study of the unit and its expansion we use as an example a symmetrical unit of four beams.

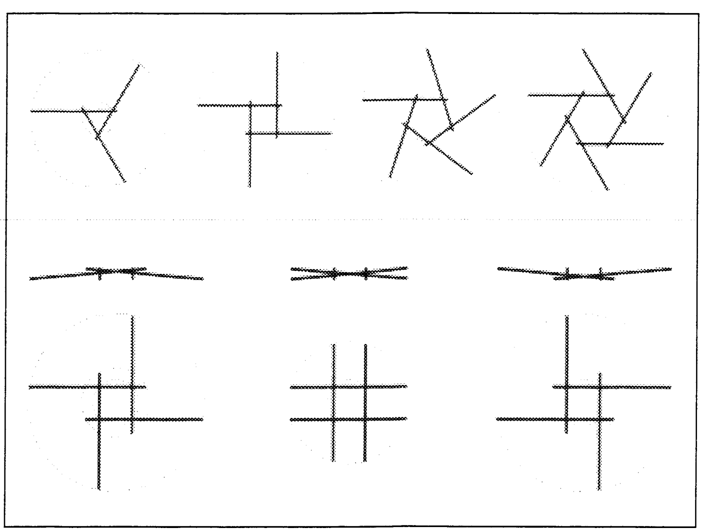

Figure 3: Table of parameters for transformations

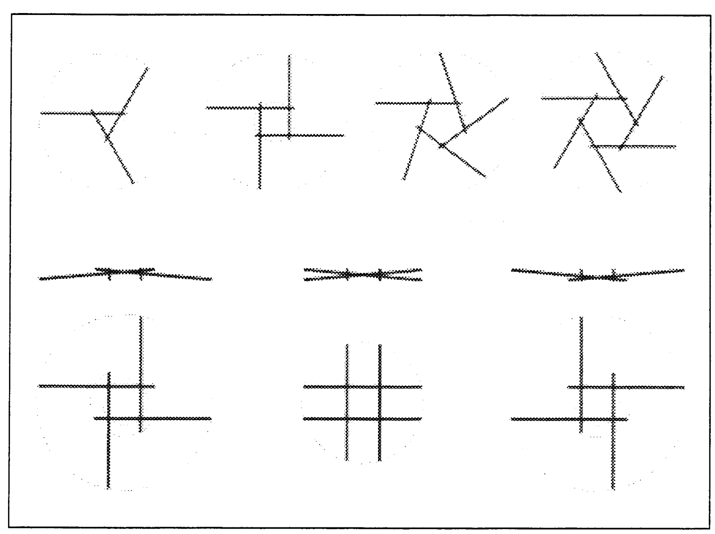

Figure 4: Stick numbers in unit / Transformation D

TRANSFORMATIONS BASED ON PARAMETRIC RELATIONSHIPS

To investigate possible forms we look at the transformations which result from changing values of parameters. This can also give an understanding of the dependencies among the parameters. If a table like in figure 3 is created in which the various parameters defined above are the rows, then the columns can represent different combinations of fixed and variable parameters; an "X" representing a parameter that is fixed and unchanging (e.g. the length of a beam segment) and an "0" indicating a parameter that changes (e.g. the diameter of the stick beams). A careful examination of each potential combination leads to the observation that some combinations are possible (indicated with a "Y" for yes) while others are not ("N" for no).

As an example, consider column D in figure 3. This combination indicates that the overall length of the stick beam (L), the length of the interior segment of the beam (L_I), the angle of inclination (a), and the stick diameter (d) are all fixed. That leaves only the lengths of the anchor and free segments (L_A and L_F), as variables. That is, they are allowed to change in order to accommodate any transformation of the basic unit form. If the position of the supported beam on each stick beam is shifted towards the ground, for instance, and the length of the interior segment is held constant, then the length of the free end will increase, as figure 4 bottom illustrates. But it can be shown that the angle of inclination will remain constant. It"s as if the stick beams are sliding through the joints, retaining their angle of inclination. The area inscribed by the stick beams keeps its shape and the basic unit retains its integrity. The position of the anchor points on the ground, however, moves closer together and one begins to notice a more prominent transformation: the basic unit changes from a "tepee" form in which the sticks are leaning together in an upright orientation, to what might be described as an "umbrella" form in which the stick beams cantilever out from a central position. This transformation has been tested with physical models and can be viewed in an animated computer model simulation.

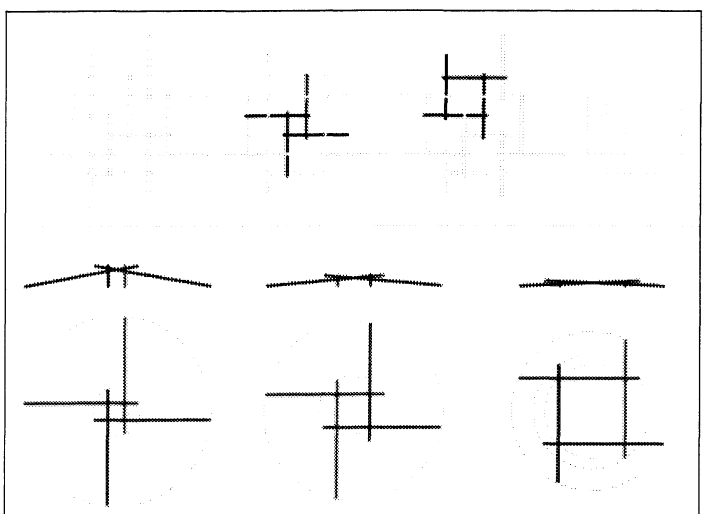

Figure 5: Perimeter expansion / Transformation E

A second transformation of the basic four member unit is represented by column E in figure 3, illustrated in figure 5 bottom. In this case the overall length (L), free length (L_F), and stick diameter (d) stay fixed while the remaining parameters, (L_A), (L_I), and (a) can vary. The resulting transformation makes the structure appear to flatten as the anchor segment length decreases, which also results in the distance between the points of support (anchor points) becoming smaller. The circumscribed square interior area increases in size like the aperture opening of a camera.

This kind of transformation recalls certain examples of kinetic or unfolding architecture. One can imagine that such structures involving the physical transformation of a basic unit could be designed if the stability of the moving structure were assured. Some studies have been made using configurations similar to those described above[8].

A final example demonstrates the effect of stick diameter on the overall shape. If we consider the parameters in column F in figure 3 (L, L_F, and a fixed; L_A, L_I and d variable), once again we discover a transformation which preserves the integrity of the basic unit while allowing its form to change. As the diameter of the stick beams is made to increase, the position of the supported beam will migrate towards the anchor end or ground provided the angle of inclination is held constant. Conversely if the position of the supported beam were held constant (that is, L_I, and thus L_A if L_F remains fixed), then the angle of inclination would have to increase as the diameter of the stick increases in order to preserve the integrity of the unit. The latter represents the transformation implicit in the parameters of column G in figure 3.

GENERATION OF COMPLEX FORMS

There seem to be innumerable possible forms of structures with more than one unit. It would therefore be interesting to categorize the forms and patterns. We make a first step by suggesting two methods which generate such patterns from a unit. But we know

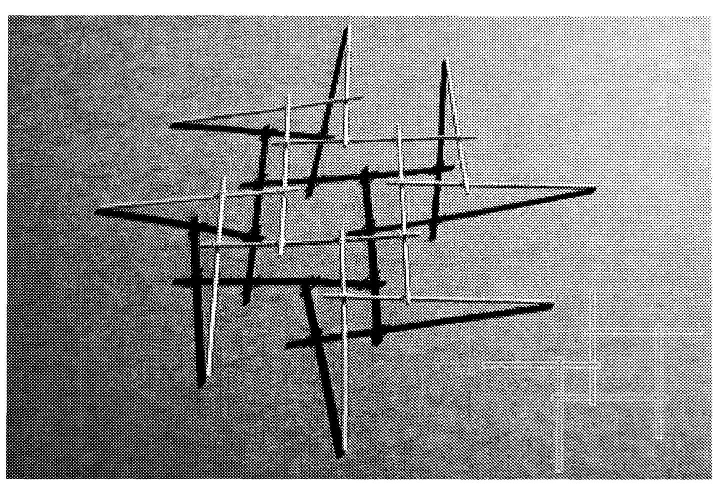

Figure 6: Model of expanded unit / Unit combination

that other categories exist which we will explore at another occasion. The two methods that will be discussed are referred to as *perimeter expansion* and *interior densification.*

Perimeter expansion describes a method that combines basic units in a simple additive way to create new and larger structures, or adds additional elements to the periphery of a structure starting with a unit so that new units are formed to which existing and new elements contribute. For example, consider four identical basic lever beam units connected together to form the composition illustrated in figure 5 top. It can be seen that for each basic unit two anchor legs previously resting on the ground are now supported in the air by the stick beam leg of another unit. Of these two, one of the stick beams must support an extra beam on what was previously the anchor length (L_A) segment of the beam. Also, one of the remaining two anchor legs must support an additional interior beam. By this process, the units are lifted causing the structure to curve up slightly more. If this procedure is continued with more units added in successive rings to the perimeter, the structure grows not only in span width but also in height, forming a shallow dome shape, as the model in figure 6 illustrates.

In the process of generating complex forms, variations of the lever beam principle emerge for some of the elements, here in all interior elements. They are illustrated in figure 2. This is because the geometric pattern requires the support of an extra beam. This will increase the amount of load and hence bending on the stick as the length has not changed. However, the amount of load may be less due to a greater distribution of load points throughout the entire structure.

Observing the pattern of expansion as illustrated in the plan view diagrams of figure 5 top, we can see a fifth unit in the centre, formed by one stick each of the original four units. This unit is entirely suspended within the structure. But apart from these units with a small square in the centre, we can also see units formed around big squares. Looking at the occurrence of two possible basic units in these structures, we can find a smallest possible expansion consisting of one of the two units and two other sticks, which complete the second unit, as the insert of figure 6 shows. And we see the pattern underlying this structure, a tiling of the plane with squares of two sizes.

Figure 7 shows how this tiling can be derived from a regular tiling of the plane with squares. By rotating and expanding the edges of the squares, these become smaller as another set of squares grows at the original nodes. Generalising this observation, it can be shown, that any tiling of the plane with convex polygons can be transformed to a pattern which can be implemented as a lever beam structure. Readers familiar with the geometry of tensegrity structures may note that an interesting correspondence can be seen between this process and the generation of a tensegrity structure from any convex polyhedron. In the case of the lever beam structure, however, the underlying pattern belongs to the geometry of the plane. But since the structure approximates a spherical surface, some geometry distortion must be absorbed within the structure.

A second method of creating more complex forms involves densification through the addition of extra members. In any given lever beam structure, additional beams can be inserted between members or added to the perimeter. In either case, the tendency will be towards increasing the density of the structure in terms of the number of members per area. Figure 8 shows such a series of densifications.

These two methods are not exhaustive, as we know other lever beam structures not falling into these two subclasses. This is one of the topics of further studies already started.

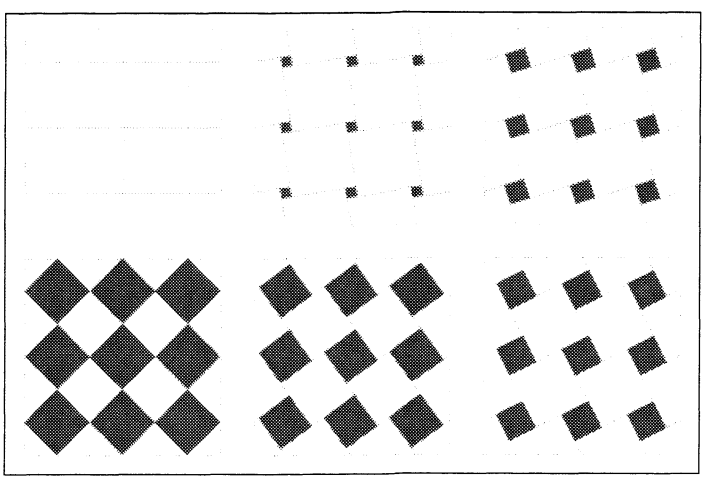

Figure 7: Generating structures from tiling patterns

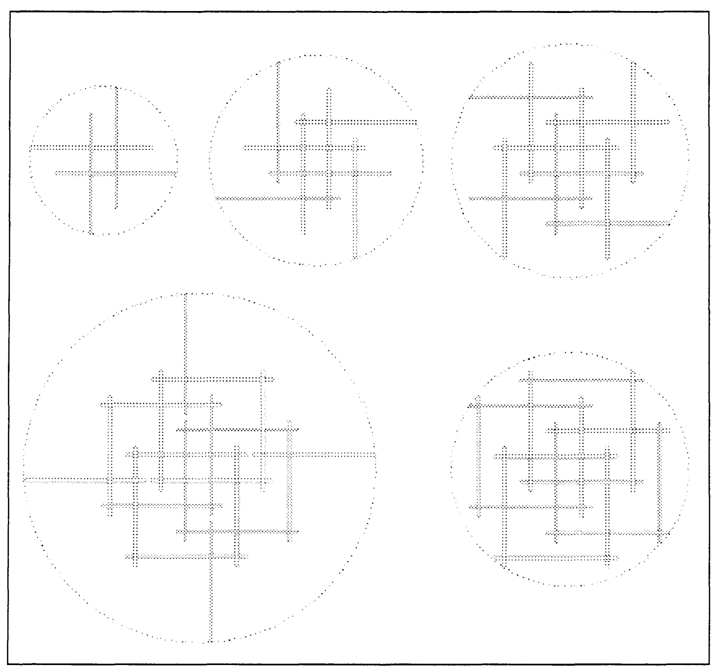

Figure 8: Interior densification sequence

BAMBOO STICK STRUCTURES

After investigating mutually supported stick structures in small-scale models to gain an understanding of the geometric and formal relationships, two large or full-scale constructions were attempted. Bamboo was used for these full-scale experiments because they are easily available in Hong Kong where scaffolding and temporary structures are still made from bamboo. Bamboo members are strong in the axial direction and possess good bending strength due to their hollow sectional shape. Since bamboo is lightweight, the construction of large structures can be accomplished by two or three persons without special bracing and temporary supports. One disadvantage of using bamboo is the variation of diameter, and the variation in strength not only between rods but within each rod. We tried to overcome this by selection of similar rods ad sorting the cut sticks for using in equivalent parts of the structure.

The structure shown in figures 9 and 10 with a span of approximately 10 meters was built using bamboo rods 1.5 m long and 4 cm in diameter. Beginning with a hexagonal basic unit, further members were added by successive perimeter additions until the structure reached the target span. Plastic ties were used on every joint to hold the members in place during construction. Upon completion it was observed that the ties were not needed for most of the joints in the upper portion of the dome where friction forces kept the bamboo sticks from sliding. On the steeper regions of the perimeter, however, the ties may have increased the friction to maintain the position of the members. The arching form of the structure caused by the accumulation of inclined lever beams resulted in a rise of about 1.7 m, about 25% less than predicted based on the small scale model studies.

Although no precision load testing or deflection measurement was attempted for this study, the bamboo dome structure was observed under the load of its own weight and a distributed loading consisting of 15 kg weights attached at 20 locations, evenly spread throughout the structure. Despite the preliminary character of the load testing, the full-scale construction none the less offered some interesting insights into the performance and constructive logic of mutually supported beam structures. First it was observed that the assembly process was straightforward and rapid, allowing the structure to be completed in just a few hours. The geometry of the structure was stable and the deadweight of the members provided enough bearing force to maintain the tightness of the overlapped joints. Second, the overall shape of the structure was regular and evenly arched attesting to the consistency of the geometric relationships. Individual members exhibited some curvature due to bending, however, this bending was evenly distributed throughout the structure. Third, the outward thrust of the anchoring stick members at the base was so small that the friction of the sticks on the grassy ground was enough to prevent movement, and no damage of the ground could be detected. Finally, failure of the system under loading occurred when the weakest member buckled due to increased bending forces. The failure created a large "hole" in the dome but did not lead to total collapse of the structure, revealing an inherent ability of the system to redistribute forces.

CONCLUSION

Following the research efforts of other investigators of reciprocal frames, this study attempted to define the characteristics and properties of a particular class of mutually supported beam structures, and to begin to understand the relationships between the geometric parameters of the system and its structural form. The principle of the lever beam and the generating pattern of the basic unit were identified as the primary "building" components of this class of structures. Employing a morphological method of analysis, parametric relationships, and the transformations they imply, were used to better understand the range of possible forms. Several cases were studied in detail using diagrams, small-scale physical models, and animated computer models. This resulted not so much in the discovery of new forms but in the uncovering of formal relationships that guide the process of transformation, from which an infinite number of form possibilities can be obtained. Finally, two full-scale structures using bamboo rods for the beams were built on an

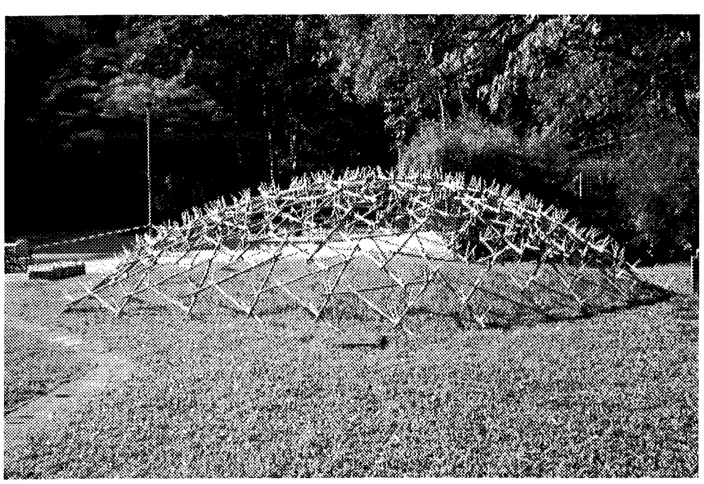

Figure 9: Overall view of built bamboo structure

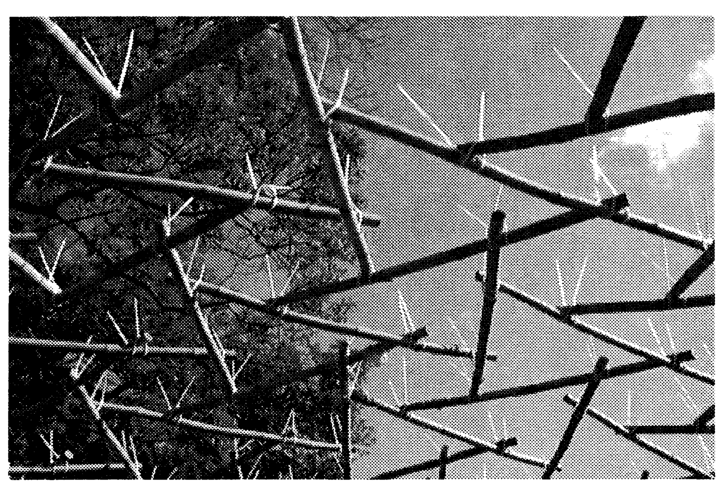

Figure 10: View of bamboo structure from inside

open site for testing the method of assembly and for observing qualitatively the behaviour and performance of the system.

Summarising the main clarifications and discoveries, we list can the following points:

- Lever beam as principle with variations
- Beams considered as elements
- Units used as building components
- Parameters of unit geometry
- One clearly defined subclass
- Plane/sphere geometry ambiguity
- Reversibility as tepee or umbrella

In addition to these conclusions, the study raised several questions that might be explored in the future. Regarding the relationship between the small scale model and full scale construction, further research needs to be made to better understand the effects of scaling up. Regarding the possible forms, we hope to still find other definable subclasses to accommodate other patterns. Further, the preliminary study of alternative beam sections and the articulation of the joints seem promising to reveal even other interesting aspects of lever beam structures.

This study has focused only on mutually supported beam structures that are three dimensional and non-directional. It is also possible to envision expansion of the basic unit in a single direction and thereby obtain a separate subclass of vault forms. There already exist historical precedents for this in the Chinese rainbow arch bridges, an example of which can be seen in a scroll painting of the twelfth century[9] of which a detail is shown in figure 11. It is interesting to compare this bridge with a sketch of a proposed bridge by Leonardo da Vinci[10], as the occurrence of a similar idea in two different cultures.

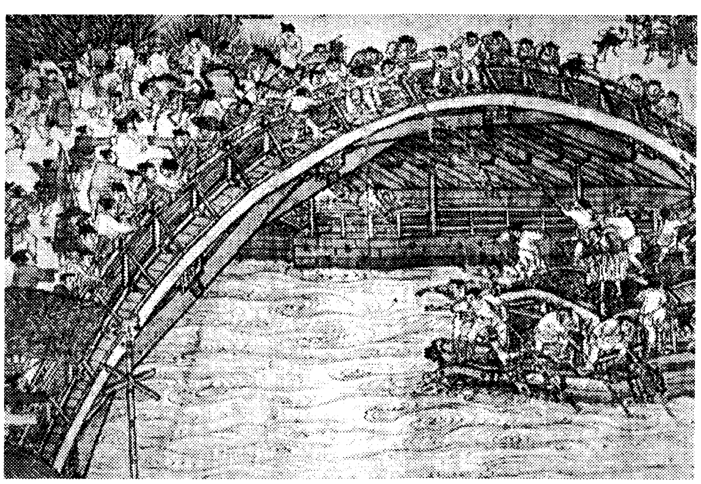

Figure 11: Partial view of Chinese rainbow bridge

NOTES

[1]See studies by John C. Chilton et al. For example, "Morphology of Reciprocal Frame Three Dimensional Grillage Structures," *Proceedings of the IASS-ASCE International Symposium*, 1994.

[2]The name "Reciprocal Frame" belongs to a system patented in the United Kingdom. It is described by Chilton as "a three-dimensional beam grillage system structural system... in which each beam in the grillage both supports and in turn is supported by the other beams in the structure (reciprocally)."

[3]Sebastiano Serlio, *The Five Books of Architecture* (New York: Dover Publications Inc., 1982).

[4]Michele Melaragno, "Trabeations: Vernacular Structures Emerging From the Historical Past," *Architectural Science Review* Volume 39 (1996): 49-57.

[5]David Yeomans, "The Serlio floor and its derivations," *Architectural Research Quarterly* Spring (1997): 74-83.

[6]*Japanese Architecture III* (London: Academy Editions, 1994).

[7]Ishii Katzuhiro, *Ishii Katzuhiro* (Tokyo: Kajima Shuppankai, 1991).

[8]J. C. Chilton, B. S. Choo, & O. Popovic, "Reciprocal Frame Retractable Roofs," *Spatial Structures: Heritage. Present and Future. Proceedings of the IASS Symposium* (1995): 467-74.

[9]Scroll painting "Qingming shang he tu" (Going up the River during the Qingming Festival) by Zhang Zeduan, 12th century as reproduced in Zhang Anzhi, *Qingming shanghe tu*, Renmin meishu chuban she, 1997. The reconstruction of the Rainbow Bridge type was the subject of a recent PBS film video (1999).

[10]The sketch by Leonardo da Vinci is found in the *Atlantic Notebook*, sheet 22 front, which is in the Biblioteca Ambrosiana, Milan. It describes a suspended beam arch bridge which bears remarkable similarity to the Rainbow Bridge. An illustration of the bridge appeared on the rear cover of *Spatial Structures: Heritage. Present and Future.* Proceedings of the IASS International Symposium, 1995.

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